



Analysis of stock exchange risk and currency in South African Financial Markets using stable parameter estimation

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Abstract

In the preceding decade, the South African economy has experienced challenges due to global disruptive events, hence, the implementation of risk mitigation strategies becomes a priority in volatile markets. Stable distributions account for skewness and heavy-tailed behaviour which are frequently observed in financial data. This study aims to investigate the fit of stable distributions for three FTSE/JSE indices and the USD/ZAR currency exchange rate. The maximum likelihood method was applied to fit Nolan's S_0 -parameterization stable distribution. Value at Risk (VaR) is measure assessing market risk, therefore, VaR estimates and Kupiec likelihood test are applied to evaluate the extreme tail behaviour of the fitted stable model. Results show the robustness of stable distributions in the long and short position for each daily returns. This research validates the use of stable distributions aimed at capturing the characteristics financial data. Those concerned with curtailing losses and investigating alternatives for financial modeling in the South African financial industry may benefit the most by using stable distributions.

Keywords: Stable distributions; Nolan's S_0 -parameterization; Stable parameter estimation; VaR; Kupiec likelihood ratio test

JEL Classifications: G13; G46; G58

Introduction

South Africa's sophisticated financial services sector consists of numerous foreign and domestic institutions that abide by a sound legislative and regulatory framework. The Johannesburg Stock Exchange (JSE Limited) is the 17th largest exchange globally by market capitalization, valued at about R6-trillion. The JSE is a vital constituent in the regulating of the South African economy. The South African financial market is a popular choice for local and foreign investors looking to explore principal capital markets within the country and across the African continent (Brand South Africa, 2017). Hence, the requirement for reliable models that track the progression of volatile indices and exchange rates and provide solutions that are useful to investors is essential - especially since disruptive events and uncertainty are frequent in the financial industry.

Stable distributions are a wealthy class of probability distributions that allow for skewness and heavy tails. This class of distributions is useful in many applications, mostly mathematical finance. Stable distributions are non-trivial limits of normalized sums of independent and identically distributed random variables. This property is suitable for modeling stock returns. The first to apply stable properties to returns of stock data is Mandelbort (1963). Four parameters describe stable distributions and allow for empirical data adaptability for testing models (Tian, 2016).

Literature suggests the common hypothesis of normality for financial data has the tendency to underestimate the probability of extreme returns, that is, fat tails and skewness. Thus, a fitted stable model where fat tails and skewness are taken into account is suggested. Stable distributions are a wealthy and valuable class of probability laws that gives a parsimonious fit to the suggested model. This paper aims to investigate the fit of the stable distributions to three FTSE/JSE market indices and the United States of American Dollar to the South African Rand using a univariate time series analysis approach. The Anderson-Darling goodness-of-fit tests justify the adequacy of the fitted stable models were applied to the daily returns of FTSE/JSE All-Share Index, FTSE/JSE Banks Index, FTSE/JSE Mining Index, and USD/ZAR exchange rate. The Kupiec likelihood ratio test is applied as a backtesting procedure to the VaR estimates with the aim of evaluating the robustness of each fitted stable model.

Fundamentally, this study aims to contribute and spotlight the usefulness of stable distributions as alternatives for modelling large sets of financial data that display heavy tails and skewness as well as attain an extensive grasp of the South African Financial sector.

Literature Review

Research by Nolan (2003) investigated the fit of stable distributions on the British Sterling Pound (GBP) versus the German Mark exchange rate returns. The data consisted of daily exchange rate for the period 2 January 1990 to 21 May 1996. The investigation used the maximum likelihood (ML) method to estimate the stable distribution parameters where the monthly currency exchange rates between the U.S. Dollar (USD) and the Tanzanian Shilling were also studied. The data observed was from January 1975 to September 1997. The study found that the USD/Tanzanian Shilling exchange rate was subject to more extreme fluctuations. The feasibility of stable parameter estimation was shown and model diagnostics display that stable distributions describe financial data well. In cases where the fit is lacking or unfitting, this study suggests a better fit than that of the Normal Gaussian model. The suitability of stable distributions in VaR calculations are emphasized.

Work by McCulloch (1997) studied the appropriateness of stable distributions with data from the stock market, specifically the stock price data known as the Centre for Research in Security Prices (CRSP). This data set was examined over forty years from January 1953 to December 1992. The goodness-of-fit was studied using graphical methods by analyzing the probability-probability and stable density plots. Diagnostics imply a good fit of stable distribution to the data.

Chinhamu et al. (2015) compared the robustness of the generalized hyperbolic and stable distributions in estimating VaR for gold price returns. Chinhamu et al. (2015) used the Anderson-Darling test, Bayesian information criterion, Akaike information criterion, and backtesting of the VaR estimates to check for model adequacy of the fitted probability distributions. The study finds that the best performing model for gold returns

differs at different VaR levels. The stable distribution and generalized hyperbolic distribution favorably describe extreme risk in gold returns.

Kallah-Dagadu (2013) evaluated three methods for estimating α -stable distributions. The ML, empirical-characteristic function, and sample quantile methods to estimate stable, normal, and Cauchy parameters for the Ghana Stock Exchange All-Shares Index, USD to Ghana Cedi (USD/GHC) exchange rate, GBP to Ghana Cedi (GBP/GHC) exchange rate and European Euro to Ghana Cedi (EUR/GHC) exchange rate. The analyzed data was for the period from 2000 to 2011. The study concluded that weekly returns of Ghanaian currency exchange rate data exhibit heavy tails and asymmetry. The maximum likelihood method produced the most precise and efficient estimates for the stable fit to the data.

Naradh et al. (2016) studied the fit of stable distributions for each of the BRICS (Brazil, Russia, India, China South Africa) currencies against the USD in both the univariate and multivariate scenarios. The data set consists of exchange rate data from each BRICS country within the period from January 2011 to January 2016. The Kolmogorov-Smirnov and the Anderson-Darling tests demonstrate that stable distributions adequately fit the BRICS financial data returns. The study evaluated the fitted models' performance in estimating VaR values using the Kupiec likelihood ratio test and the Christoffersen's conditional coverage test. This study brings to the fore the usefulness of stable distributions in estimating VaR of BRICS financial data.

Borak et. al. (2005) emphasizes the empirical evidence by Fama (1965) and Mandelbrot (1963) that suggests stable distributions as a heavy tailed alternative. Stable distributions allow for asymmetry and fat tails and are beneficial models in view of extreme events such as the global market crisis or natural calamities. Empirical evidence shows a robust fit of stable laws for DJIA index and Boeing stock.

Work by Pele (2012) analyzed the BET Bucharest Stock exchange where the likelihood of extreme events was estimated by stable distributions. The paper highlights that fitting returns to the Normal distribution is useful however the model fails to consider the probability of extreme events. The study has further showed that stable distributions improve predicting an extreme event.

There is limited research on the topic of modelling JSE indices and the USD/ZAR exchange rate to stable distributions to the best of our literature knowledge. The main contribution of this study is to investigate the soundness of the stable distribution in estimating VaR values for the JSE financial data and USD/ZAR currency exchange rate that exhibit heavy tails and skewness.

Research and Methodology

In order to test the hypothesis, this study discusses and applies the S_0 -parameterization by Nolan (2020). Nolan (2020) describes the S_0 -parameterization as:
 A random variable Y is $S(\alpha, \beta, \gamma, \delta; 0)$ if

$$Y \stackrel{d}{=} \begin{cases} \gamma \left(Z - \beta \tan \frac{\pi\alpha}{2} \right) + \delta, & \alpha \neq 1; \\ \gamma Z + \delta, & \alpha = 1. \end{cases} \quad (1)$$

where $Z \equiv Z(\alpha, \beta)$ is the characteristic function. In this case Y has characteristic function:

$$E \left(e^{itY} \right) = \begin{cases} \exp \left(-\gamma^\alpha |t|^\alpha \left[1 + i\beta \left(\tan \frac{\pi\alpha}{2} \right) (\text{sign}(t)) \times (|t|^{1-\alpha} - 1) \right] + i\delta t \right), & \alpha \neq 1; \\ \exp \left(-\gamma |t| \left[1 + i\beta \frac{2}{\pi} (\text{sign}(t)) \times \log(\gamma |t|) \right] + i\delta t \right), & \alpha = 1. \end{cases} \quad (2)$$

Nolan (2020) recommends using the S_0 -parameterization for statistical inferences, and numerical purposes, as it has the simplest form for the characteristic function that is continuous in all four parameters. The S_0 -parameterization acknowledges a location-scale family. If $Z \sim S(\alpha, \beta, \gamma, \delta; 0)$, then for $\alpha \neq 0, b \in R, aZ + b \sim S(\alpha, \text{sign}(\alpha)\beta, |a|\gamma, a\delta + b; 0)$.

Y has characteristic function:

$$E\left(e^{i\gamma Y}\right)=\begin{cases} \exp\left(-\gamma^\alpha|t|^\alpha\left[1-i\beta\left(\tan\frac{\pi\alpha}{2}\right)\left(\text{sign}(t)\right)\right]+i\delta t\right) & \alpha\neq 1; \\ \exp\left(-\gamma|t|\left[1+i\beta\frac{2}{\pi}\left(\text{sign}(t)\right)\log(\gamma|t|)\right]+i\delta t\right), & \alpha=1. \end{cases} \quad (3)$$

Stable parameter estimation

Nolan (2020) emphasizes that several standard parameter estimation procedures fail to work for stable data since there is a lack of closed-form densities for stable distributions. No one method is considered the superior or most efficient, however, the maximum likelihood method is the most commonly used method applied for stable parameter estimation. This study investigates the accuracy of the maximum likelihood (ML) estimation method in stable parameter estimation.

Anderson-Darling (AD) test

The Anderson-Darling goodness of fit test was the result of extensive research by T.W. Anderson and D.A. Darling (Anderson & Darling, 1952).

The Anderson-Darling test statistic is A^2 defined as

$$A^2=-m-\frac{1}{m}\sum_{i=1}^m(2i-1)\left(\ln(\hat{F}(x_{(i)}))+\ln(1-\hat{F}(x_{(m+1-i)}))\right) \quad (4)$$

where $x_{(1)}<...<x_{(m)}$ is the ordered sample size m from smallest to largest and $F(x)$ is the underlying theoretical cumulative distribution to which the sample is compared.

The null hypothesis $\{x_{(1)}<...<x_{(m)}\}$ comes from the underlying distribution of $F(x)$ is rejected at a specified level of significance (α), if the test statistic A^2 is greater than the critical value for a table of critical values at different sample sizes. Generally, critical values of the Anderson-Darling test statistic depend on the distribution being tested. The goodness-of-fit of several distributions may be evaluated using the Anderson-Darling test.

Value-at-Risk and Backtesting

One of many implications of Basel Committee on Banking Supervision's creation was the implementation of Value-at-Risk as the standard benchmark measure for evaluating market risk. The capital requirements of financial institutions are based on VaR estimates. Therefore, tests for evaluating the out-of-sample forecast accuracy of the VaR model through backtesting procedures have become of vital practical importance (Escanciano & Olmo, 2010). VaR aims to evaluate the maximum possible loss for a portfolio over a specified period, and its VaR calculations focus on the tails of a distribution. This provides procedures for testing the robustness of a model. For a random variable Y , which is usually the log-return of a risky financial instrument with distribution function F over a specified period, VaR at given probability p is defined as the p -th quantile of F , that is,

$$\text{VaR}_p=F^{-1}(1-p) \quad (5)$$

where F^{-1} is the quantile function.

To examine the effectiveness and adequacy of VaR, various backtesting procedures are utilized. Formal conclusions on model robustness can be obtained using the Kupiec likelihood ratio test and Christoffersen conditional coverage test (Kupiec, 1995).

Empirical Data and Analysis

Data Exploration

This study's data sets are the daily FTSE/JSE All-Share Index, FTSE/JSE Banks Index, FTSE/JSE Mining Index and USD/ZAR prices obtained from McGregor BFA and were noted over the period from 13 August 2010 to 14 August 2020. The return series for each index is calculated as the first backward differences of the index values' natural logarithm. For day t , the daily log return r_t is defined as

$$r_t = \ln(P_t) - \ln(P_{t-1}) \quad (6)$$

where P_t is the price at day t .

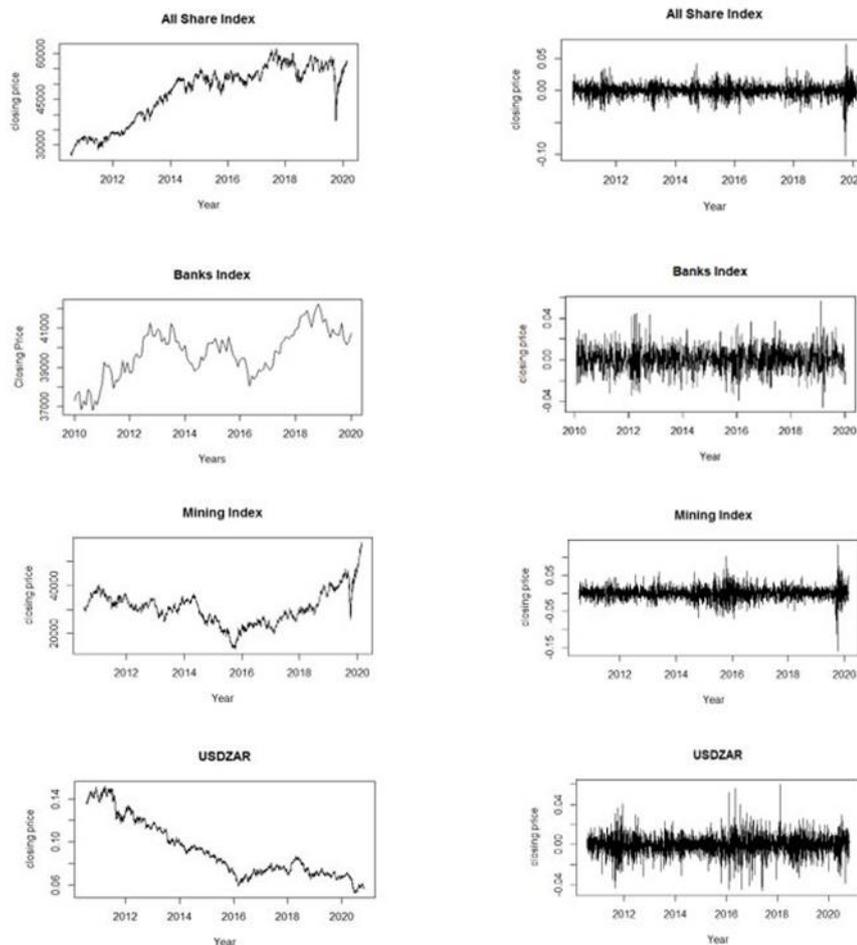


Figure 1: Time series plot of JSE Indices and USDZAR exchange rate (left) and one day returns (right)

In Figure 1, the plots specify numerous trends in mean and variance over time indicating non-stationarity. The log returns are stationary as the mean fluctuates around 0, however, the variance varies over time indicating heteroscedasticity and volatility clustering which is probable when dealing with financial data. Isolated extreme returns caused by shocks to financial markets are evident, such as the 2015 stock market crash and the 2019-2020 global COVID-19 pandemic.

Table 1: Descriptive statistics of financial market indices and exchange rate price returns

	ALSI		Banks Index		Mining Index		USD/ZAR	
Panel A Descriptive Statistics								
No. of observations	2499.00		2499.00		2499.00		2675.00	
Minimum	-0.1023		-2.3021		-0.1589		-0.0460	
Maximum	0.0726		0.0991		0.1346		0.0603	
Mean	0.0003		-0.0008		0.0002		-0.0003	
Median	0.0006		0.0005		0.0003		0.0000	
Skewness	-0.7310		-41.0875		-0.1605		-0.1671	
Excess Kurtosis	8.8822		1919.8771		6.0443		2.7766	
Panel B Testing for normality, autocorrelation and heteroscedasticity								
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
Jarque-Bera	8455.2667	<0.0001**	385117132	<0.0001**	3823.93	<0.0001**	874.3992	<0.0001**
Ljung Box	67.2900	<0.0001**	7.0024	0.9967	47.62	0.0005	11.7751	0.9236
ARCH LM Test	936.0966	<0.0001**	0.00	0.99	50.03	0.00	15.41	0.00
Panel C Testing for unit root and stationarity								
	Statistic	p-value	Statistic	p-value	Statistic	p-value	Statistic	p-value
ADF Test	-13.6259	0.01	-12.9576	0.01	-13.5509	0.01	-14.7402	0.01
PP Test	-2586.908	0.01	-	0.01	-	0.01	-	0.01
KPSS Test	0.1303	0.10	0.0914	0.10	0.2394	0.10	0.0431	0.10

Descriptive Statistics for the daily closing prices of the FTSE/JSE financial stock indices returns and USD/ZAR are shown in Panel A of Table 1. Positive mean averages for ALSI and Mining Index indicate a slight growing trend over time whereas the negative averages for the Banks Index and USD/ZAR indicate a slight declining trend over time for the return series. The excess kurtosis value indicates the leptokurtic behaviour of these return series. This means that the empirical distribution of the daily returns is much fatter than the popular normal distribution. Large values for skewness and excess kurtosis are noticed for the Banks Index returns. This may be on account of the poor performance of the South African economy accompanied by the severe rate of unemployment as well as the government bail out of the state power utility (Eskom Holdings SOC Ltd) has proven to have negative effects for South African Bank stocks Changole (2019). The Jarque-Bera test for normality gives a p -value less than 0.0001 for all four returns, thus rejecting the normality assumption at all levels of significance. Panel B shows tests for normality, autocorrelation and heteroscedasticity are shown. The null hypothesis of normality for the Jarque-Bera test is rejected at 5% level of significance for all stock and currency returns. This infers considering the use of heavy tailed models when analyzing the returns series.

The significant p -values of the Ljung box test for ALSI and the Mining Index suggest rejecting the null hypothesis of no autocorrelation. Conversely, the null hypothesis for FTSE/JSE Banks Index and USD/ZAR exchange rate is rejected implying that the return series show serial correlation. This test provides mixed results.

Results for the unit root and stationary tests are displayed in Panel C. At a 5% level of significance, the null hypothesis of a unit root is rejected, and it can be decided that all return series are stationary. The KPSS test showed that all returns are stationary since all p -values are 0.1 which is greater than 0.05 therefore the null hypothesis of stationarity is rejected.

Results and Discussion

Parameter Estimation

Stable parameters are estimated under Nolan's S_0 - parameterization using maximum likelihood estimation. Table 2 reports the values estimated for each stable parameter on the returns of the financial data under investigation. The results are presented and discussed in the subsequent sections. Nolan's $S_0(\alpha, \beta, \gamma, \delta)$ univariate stable distributions is fitted to the daily stock returns of each market index and the USD/ZAR exchange rates.

Table 2: Stable parameter estimates under Nolan's $S_0(\alpha, \beta, \gamma, \delta)$

Financial Stock return				
Stable Parameter Estimates				
	ALSI	Banks Index	Mining Index	USD/ZAR
$\hat{\alpha}$	1.7316	1.7522	1.7400	1.7729
$\hat{\beta}$	-0.2464	-0.0894	<0.0002	-0.3250
$\hat{\gamma}$	0.0060	0.0097	<0.0002	0.0059
$\hat{\delta}$	0.0008	0.0004	<0.0002	0.0002

The skewness parameter β is negative for all returns except the Mining Index therefore indicating that the fitted univariate stable distribution is skewed to the left. Since location parameter, $\delta > 0$ for all returns it can be implied that the fitted distributions have a rightward shift.

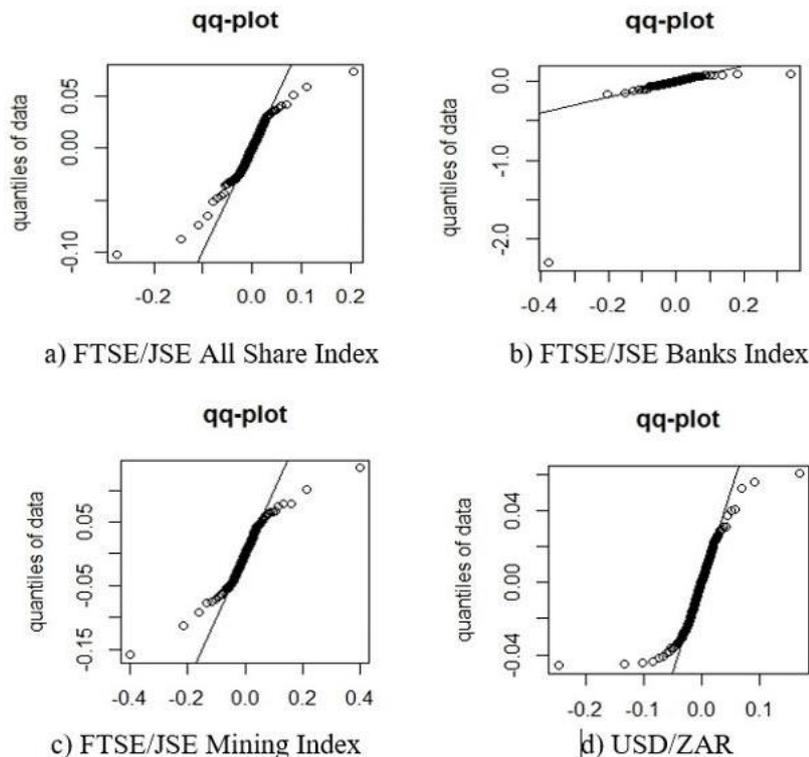


Figure 2: Q-Q plots for financial market indices and exchange rate returns

In Figure 2, the Q-Q plots seem to be visually compressed with extreme values dominating the plot. The heavy tails, evident in these Q-Q plots, show that the extreme order statistics have a lot of variability. Therefore, deviations from the ideally straight-line Q-Q plot are difficult to assess. This also shows that extreme tails from the data set are lighter than the stable model (Nolan, 2003). The Q-Q plots indicate the

inadequacy of the stable model at extreme values. Therefore, the problems mentioned about the Q-Q plots lead us to focus on formal model adequacy test such as the Anderson-Darling goodness-of-fit test.

Table 3: Anderson-Darling goodness of fit test for daily returns

Financial Stock return				
Stable Parameter Estimates				
	ALSI	Banks Index	Mining Index	USD/ZAR
AD Test	0.6649	0.4813	0.4025	1.6934
(p-value)	((0.5888)	(0.7659)	(0.8462)	(0.1364)

The Anderson-Darling test from Table 3 shows that the fitted stable models are significant, therefore, it is concluded that the fitted $S_0(\alpha, \beta, \gamma, \delta)$ model is adequate in describing for each financial market indices and exchange rate.

Stable density plots

Empirically, the density plots of the daily stock returns and the fitted univariate S_0 stable distribution are compared. Figure 3 shows a good fit of the estimated stable model to the data's daily returns as the fitted stable model does not deviate much from the returns of each financial index and exchange rate returns. A close fit for the data is given over most of the range, with extreme tails being overestimated.

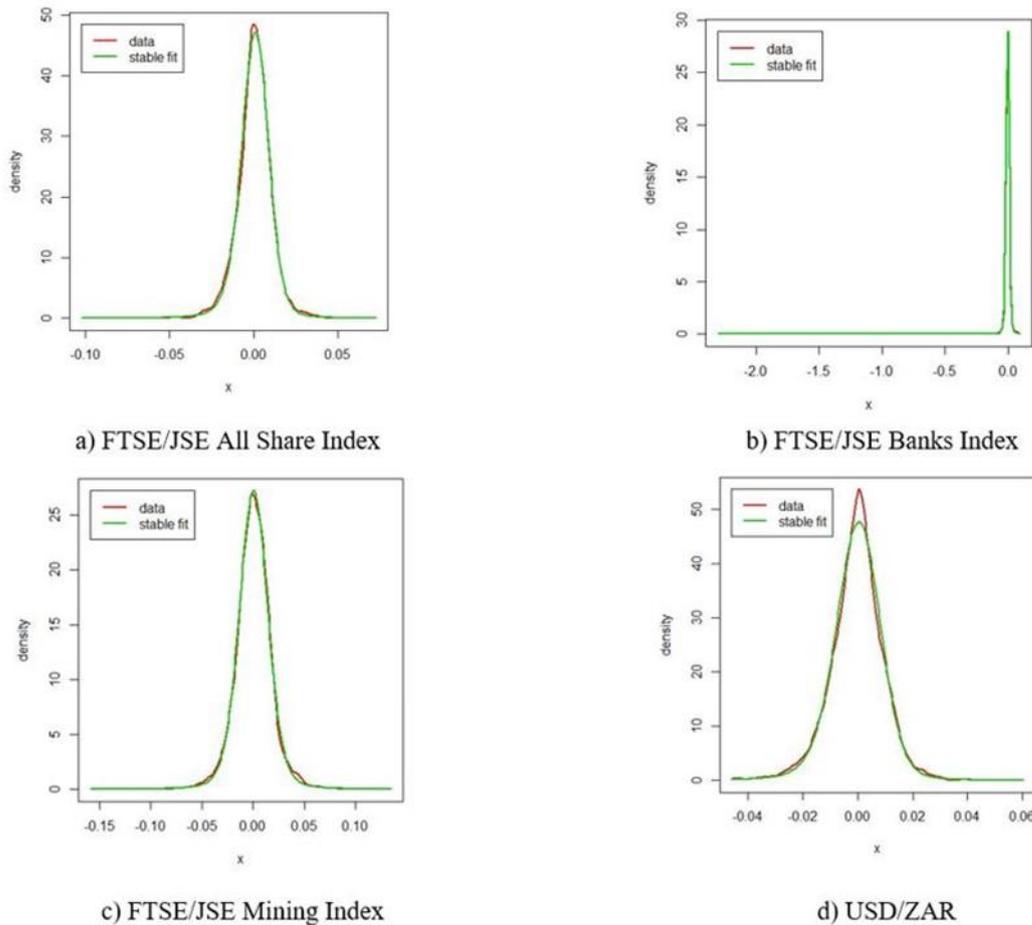


Figure 3: Stable density plots Financial market indices and exchange rate

VaR and Backtesting

Table 4: VaR estimates of financial market indices and exchange rate price returns using fitted stable model

		VaR level					
		Short position			Long position		
		1%	2.5%	5%	95%	97.5%	99%
Fitted S_0 -stable model	ALSI	-0.0317	-0.0212	-0.0158	0.0154	0.0193	0.0264
	Banks Index	-0.0466	-0.0324	-0.0250	0.0246	0.0314	0.0437
	Mining Index	-0.0491	-0.0344	-0.0266	0.0271	0.0349	0.0497
	USD/ZAR	-0.0301	0.0209	-0.0160	0.0142	0.0177	0.0233

Table 5: Kupiec p -values for financial indices and exchange rate returns

		VaR level					
		Short position			Long position		
		1%	2.5%	5%	95%	97.5%	99%
Fitted S_0 -stable model	ALSI	0.0300	0.9464	0.4644	0.7115	0.2849	0.5527
	Banks Index	0.5395	0.4765	0.2758	0.2758	0.6544	0.6900
	Mining Index	0.5395	0.5665	0.5212	0.9233	0.3440	0.2986
	USD/ZAR	0.2456	0.7009	0.4169	0.9118	0.3855	0.6662

VaR estimates for the FTSE/JSE Indices and USD/ZAR are seen in Table 4 and associated Kupiec p -values are found in Table 5. From Table 5, it is observed that at a 5% level of significance, the Kupiec test indicates that the fitted stable model is a suitable fit at almost all VaR levels for each of the returns since the p -values are greater than 0.05. Thus, the null hypothesis of model adequacy is not rejected. The p -value for the All Share Index may indicate model inadequacy at a 5% level of significance for the 1% VaR probability level. However, at a 1% level of significance, the fitted stable model is a fairly good fit.

Discussion

Modelling in the financial industry has for a long time followed the assumption that financial returns follow a Normal distribution. The Gaussian paradigm comprises of many favourable analytical properties which are similar to members of the stable distribution family (Yang, 2012).

Reasons why the Normal distribution are accepted in financial modeling are:

- It is a simple and practical distribution where numerical methods can be implemented
- Normally distributed random variables assume values around the central mean where as one moves away from the mean the odds of deviation exponentially decrease
- The Central Limit Theorem and the Law of Large Numbers are properties that reduce complexities or problems in Statistics by working with distributions that are approximately Normal. Stoyanov et al. (2011) mentions the Black-Scholes option pricing model, Capital asset pricing model and Markowitz's modern portfolio theory as notable financial modelling frameworks.

Empirical analysis by Pele (2012) on the Bucharest Stock Exchange highlights that the Normal distribution underestimated the probability extreme events where as the application of stable models greatly improves predictions of extreme events. Jama (2009) rejects the Gaussian model with evidence from the Johannesburg Stock Exchange (FTSE/JSE) and notes more reasonable VaR and Conditional VaR for stable models than that of the Gaussian distribution. There are numerous studies with empirical evidence that suggest the Normal distribution fails to adequately capture extreme returns often observed in financial markets. Thus, in order to overcome the shortcomings and inadequacies of the Gaussian approach, modelling financial asset returns using the family of stable models is suggested as a better alternative.

Initial descriptive statistics tests were done to determine the underlying nature of each return series. Each of the daily log returns of the three FTSE/JSE indices and the Dollar/South African exchange rate has shown to be non-stationary, with evidence of heteroscedasticity and volatility clustering. The Jarque-Bera test for normality rejects the normality assumption thus reinforcing the inadequacy of the Gaussian approach as mentioned by the work of Pele (2012) and prompts the subsequent fitting of stable distributions as an alternative. The fitting of a statistical distribution usually presumes no serial correlation and heteroskedasticity. However, the empirical properties of financial returns as recognized by McNeil et al. (2005) describes that some returns in financial data show serial correlation, this is the case with FTSE/JSE Banks Index and USD/ZAR where serial correlation is noticed. The maximum likelihood method was used to estimate stable parameters under Nolan's S_0 -parameterization and Q-Q plots of each return series highlight the shortfalls of fitting the stable models at extreme values thus leading to more formal good of fit tests, namely as the Anderson-Darling goodness-of-fit test. Results from the Anderson-Darling test confirm the suitable fit of stable distributions for all return series. VaR estimates were calculated and the Kupiec likelihood results show robustness of the stable model at most VaR level thus emphasizing the use of fitting stable models when describing South African financial data where fat-tails and asymmetry are prevalent.

Conclusions

The daily log returns of 3 FTSE/JSE indices and the Dollar/South African exchange rate (FTSE/JSE All Share Index, FTSE/JSE Banks Index, FTSE/JSE Mining Index and USD/ZAR) were analyzed using the fitted univariate Nolan's S_0 - parameterization stable distribution. This study substantiates the results of Nolan (2020) that stable distributions are a flexible class of probability laws that can sufficiently capture the characteristics of South African financial data. This paper shows that the estimation of stable parameters is suitable, and diagnostics prove that financial data of considerable size with heavy tails and skewness are represented well by stable models as confirmed by the Anderson-Darling goodness-of-fit test.

VaR estimates and VaR in-sample backtesting using the Kupiec likelihood ratio test point out the robustness of the fitted stable models. The use of stable distributions for data in finance is largely justified in this study by capturing fluctuations as seen in the time series plots suggests the need for better models is essential for acknowledging the many empirical properties of financial data. Policymakers, regulators, risk-averse investors, and insurers could gain the most using stable distributions as they are parties that remain chiefly concerned about extreme losses. As further research, the comparisons to other distributions such as the Generalised Pareto distribution (GPD) or a stable-GARCH mixed model comparison with thorough tail behavior evaluations and with the use of Value-at-Risk (VaR) estimates and backtesting are suggested to evaluate model performance. Furthermore, this paper highlights stable distributions as a possible alternative than the traditional Gaussian approach for financial modelling in South Africa.

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